MITIGATION OF FREQUENCY OSCILLATIONS IN A SYNCHRONOUS MACHINE INFINITE BUS SYSTEM WITH LOW MOMENT OF INERTIA

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Abstract
Integration of Renewable energy sources (RES) to power grids has led to undesirable effects to power systems Stability. Since RES such as wind and solar energy provide low to none moment of inertia, power systems are more vulnerable to power impacts, sudden load changes, and electrical faults. Systems with low moment of inertia when subjected to disturbances experience greater rates of change of frequency (Rocof) at a faster pace than traditional systems, leading to higher risks of damaging mechanical vibrations to generators, fault cascades, and blackouts. The objective of this research is to analytically characterize frequency oscillations in a synchronous machine infinite bus (SMIB) system when arisen from small perturbations and propose a method to mitigate them by using techniques from optimal control theory.

Keywords
Small signal Stability; frequency deviations; Integrating Renewables; optimal control
INTRODUCTION

During many years, power systems operation has been completely based on generation by means of thermal, hydro, and nuclear power plants. This kind of generation is based on large heavy rotating machinery that aids system stability. These generators provide high rotational inertia, through their stored kinetic energy, a fundamental property for power systems frequency dynamics and stability [1]. In case of frequency deviations, synchronous generators’ rotating mass releases energy to the grid. Thus, mitigating variations in frequency. According to the swing equation, frequency deviations are inversely proportional to two times the inertia constant H and proportional to power variations.

\[
\frac{d\Delta f}{dt} = \frac{1}{2H}(\text{Power}_{\text{generated}}(t) - \text{Power}_{\text{consumed}}(t)) \quad (1).
\]

Operating at a high inertia constant, H, allows for a more benevolent frequency dynamics, slower Rociof. Thus, enhancing grid reliability. Keeping grid’s frequency within an acceptable range is an optimal scenario for steady state operation [1]. Therefore, frequency stability depends upon active power balance, meaning that power generated minus power demanded must be kept approximately at zero. Nowadays, renewable energy resources have greatly altered power systems total inertia constant H since this type generation contribute low to none moment of inertia to the overall system inertia, raising the question of how severe system frequency oscillations could be under low inertia scenarios.

The swing equation establishes that the product of the moment of inertia J and the angular acceleration with respect to a stationary reference frame is equal to the change in torque producing it. For convenience, this mechanical equation is translated into electrical variables in a rotating reference frame to model the dynamics of synchronous generators [2]. This second order differential equation can be further broken down into two first order nonlinear differential equations. Typically, nonlinear differential equations are studied by finding it’s solutions numerically. Nevertheless, since power impacts are usually small in magnitude, linearization around an operating point is allowed and therefore their study can be carried out by finding the eigenvalues, poles, associated to the system’s characteristic polynomial. The two equations describing the SMIB system’s motion are:

\[
\frac{d\Delta \omega_r}{dt} = \frac{1}{2H}(T_s - T_d \Delta \delta - T_d \Delta \omega_r) \quad (2).
\]

\[
\frac{d\delta}{dt} = \omega_0 \Delta \omega_r \quad (3).
\]

Where Ts, the synchronizing torque component, is in phase with the angle deviation \(\Delta \delta\). Lack of synchronizing torque leads to non-oscillatory instability; the damping torque Td is in phase with speed deviations, lack of Td leads to oscillatory instability. Writing the above equations in Laplace transform; manipulating eq (3) in terms of eq (4), and counting for damping yields the following:

\[
S^2(\Delta \delta) + S \frac{K_d}{2H} (\Delta \delta) + \frac{K_i \omega_0}{2H} = \frac{\omega_0}{2H} \Delta T_m \quad (4).
\]

\[
[\begin{bmatrix} \Delta \delta \\ \Delta \omega_r \end{bmatrix}] = \begin{bmatrix} 0 & -1 \\ -w_n^2 & -2\xi w_n \end{bmatrix} [\begin{bmatrix} \Delta \delta \\ \Delta \omega_r \end{bmatrix}] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Delta T_m \quad (5).
\]

Where eq (5) is the state space representation of eq (6). For the sake of this investigation, the effects of low moment inertia on Rocof is analyzed using eq (6) while the Heffron-Phillips model is discussed for finding optimal eigenvalues in low inertia scenarios that include automatic voltage regulation.
MATERIALS Y METHODS

Throughout this analysis, the computational package MATLAB has been extensively used to run simulations and designing controllers using the control toolbox commands. Because the damping ratio $\xi$, a measure that describes how fast system oscillations decay after a disturbance, plays a critical role in describing transient phenomena, our focus is to examine it as a function of varying values of $H$.

$$\xi(H) = \frac{K_d}{2\sqrt{\omega_0 K_s 2H}} \quad (6)$$

By taking its derivative with respect to $H$, a better insight must be gained about Rocof as $H$ varies. On the other hand, the natural frequency of oscillation dictates how a system damps out oscillations without any external damping force

$$\frac{d\xi}{dH} = \frac{K_d \omega_0 K_s}{2(\omega_0 K_s 2H)^{3/2}} \quad (7) \quad ; \quad \frac{d\omega_n}{dH} = -\frac{\omega_0 K_s}{(2H)^{3/2}} \quad (8)$$

Since the derivative of $\xi(H)$ with respect to $H$ holding $K_d, \omega_0$, and $K_s$ constant is negative, the damping ratio function decreases as $H$ varies. This result can be interpreted as the less inertia in the system, the greater the damping ratio and hence a faster decaying of oscillations can be expected. Furthermore, the derivative of the natural frequency with respect to $H$ is also negative which implies that lower levels of inertia yield greater natural frequencies of oscillation. This two results might appear inconsistent. On the one hand, higher damping promotes quicker settling times while on the other hand, low values of $H$ lead to more severe vibrating frequencies.

RESULTS AND DISCUSSION

Figure 1 shows frequency oscillations to different values of $H$ in a SMIB system when subjected to a power impact caused by a sudden increment of 0.35 in mechanical torque. As expected, the variations in frequency that experienced the most severe oscillations are the ones with the lower inertia values while the ones with higher values of $H$ exhibit a smoother and slower behavior.

![Figure 1: frequency oscillations to different values of H and their corresponding eigenvalues](image-url)
In contrast to the general belief that left most eigenvalues imply a more stable dynamical system, these eigenvalues represent a quick convergence, desiring equilibrium is approached faster, yet it represents a less robust system. If poles of this system were to move farther into the left half plane, the nonlinear system might not act exactly like the linear system previously considered. Consequently, there must be an ideal set of eigenvalues that can be placed to best trade-off fast performance, avoiding damaging frequency overshoot, and cost of control. Given a $\dot{x} = Ax + Bu$ system and if $A$ is a controllable and observable matrix, then there must be an actuator $u = -kx$ where $k$ is the set of optimal gains that yields the most desirable set of eigenvalues of the system $\dot{x} = (A - Bk)x$.

This optimal theory of eigenvalue placement is achieved by a Linear quadratic regulator. A linear quadratic regulator defines a set of optimal eigenvalues based on two cost functions. The LQR minimizes a quadratic performance consisting of state and control matrices penalizing performance and cost [3].

$$J = \int_{t_0}^{t_f} (x'Qx + u'Ru) dt \quad (9)$$

In eq (7) $Q$ is an $n \times n$ matrix, $n$ being the number of state variables, that penalizes states for not reaching equilibrium fast and $R$ is the cost of using an actuator $u$ [3]. For instance, since renewable energies require more complex control strategies when subjected to disturbances, supercapacitors or battery banks are used as synthetic inertia to prevent frequency oscillations. These types of technologies require more energy which in turn increases the cost of actuators, $R$ value.

**FIGURE 2:** Control system design using pole placement. Block diagram representation

**FIGURE 3:** Frequency oscillations using an LQR controller

**FIGURE 3:** Frequency oscillations without using an LQR controller
Figure 3 shows the proposed control strategy, where an LQR controller is only used when low inertia values are exhibited in the SMIB system. In this example, the case study in Figure 1 is used to test the effectiveness of the LQR controller design. From Figure 3, it can be shown that systems with low values of H, 2.5 to 4, display a similar frequency behavior than the systems with higher values of H, clearly proving the validity of the controller design. In addition, the proposed strategy has been tested on SMIB systems where automatic voltage regulators are implemented. One of the AVR’s effect is to increase the synchronizing torque component and decrease damping torque when a certain gain is set negative [2]. Negative damping torque compromises power system stability since less damping implies greater oscillations. Using the MATLAB’s command LQR (A, B, Q, R) to design our LQR controllers it can be shown that frequency oscillations are greatly reduced in systems with negative damping. Figure 4 shows different frequency behaviors from different inertia constant H, 2.5 to 7, subjected to a small perturbation in systems displaying negative damping. Finally, for further research, a comparison between an LQR controller and traditional power system stabilizers is proposed to observe differences and similarities in performance.

**CONCLUSIONS**

It can be concluded that LQR controllers greatly mitigate frequency oscillations in SMIB systems with low inertia.

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**REFERENCES**

